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The Conventionality of Simultaneity

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1. Introduction

There is an honourable tradition in the philosophy of time that sees simultaneity as an equivalence relation defined on the set of all actual and possible events[1] and time itself as the linearly ordered quotient set induced by this equivalence relation. The equivalence classes of simultaneous events provide successive boundaries between a 'fixed' past where present action can produce no change and an 'open' future, a limbo of unrealized possibilities, the 'moving' present breathes momentary reality on events, before they are ~~confined~~ consigned to ~~another~~ the other limbo of dead facts. Such is the rhetoric of A-theorists as opposed to B-theorists, who take the first part of this analysis with its two-place predicate of temporal precedence ordering the simultaneity classes, but reject the ontological role of the moving present. Classical physics with its notion of absolute time provided uncritical support for the first part of the analysis, and while not actually requiring the ~~A-theory~~ additional A-theory role for the 'present' simultaneity class was certainly not inconsistent with it. The object of the present paper is to

import

consider the role (SR) of the special theory of relativity as putatively undermining classical notions which it must be admitted reflect the unsophisticated intuition of the ~~our~~ legendary friend, the Man on the Clapham omnibus!

new para

There are three quite different versions of how we should understand the notion of simultaneity in SR.

(2) SR replaces the notion of simultaneity as a two-place relation between events with a three-place relation signifying simultaneity with respect to an inertial frame.

(1) SR gives up the notion of simultaneity as an equivalence relation, and hence (with) transitivity and replaces the classical conception by what Grünbaum somewhat confusingly calls 'topological simultaneity', two events being 'topologically' simultaneous if each lies within the light cone centred at the other.[2]

(3) The classical notion still survives, since only one of the relativized notions introduced in (2) is 'real' simultaneity

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We shall now discuss these three approaches in more detail and address the various arguments that have been produced in favour of each

As we shall see, it is the ~~second~~ first, that has led to the notion of the conventionality of ~~that~~. Grünbaum refers to as metrical simultaneity.

2. Option (1): Topological simultaneity

The classical arguments in the literature. for this position are due to ~~Eynstein~~ Reichenbach and Grünbaum[3].

Reichenbach's argument is essentially as follows. How could we tell that two spatially separated events were simultaneous? We have two identically constructed clocks situated at A and B. How can we check that when the A-clock reads a time T_A, then at the self-same time the B-clock reads T_B which is numerically the same as T_A? We must in some way communicate between A and B so as to establish the required synchrony. Suppose we send a signal from A to B with a velocity v.

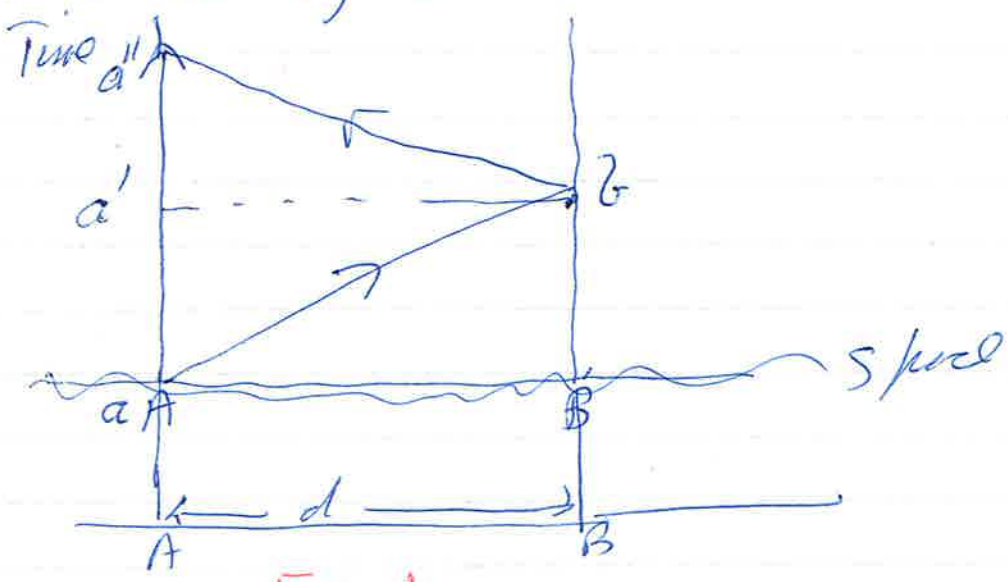

Fig. 1.

at leaves A at spacetime point a, and reaches location B at spacetime point b, as shown in Fig. 1.

If d is the spatial separation of A and B, then clearly we must set our clock at B so that the time reading t_b at b is numerically equal to the time reading t_a' for ~~clock~~ A the A-clock, where t_a' is later than t_a, the time at which the signal is sent by an amount d/v. But Reichenbach, this method will only work if we can measure v. But that always requires us to measure both of time across a spatial interval and hence presupposes the very notion of synchrony we are trying to establish. But the situation is not totally hopeless. Let the signal return from b to a". Then t_b must lie between t_a and t_a", otherwise the signal would not be propagating forwards in time. The (ta, ta") can be measured just by the A-clock, so therefore the interval does not presuppose any knowledge of synchrony between locations A and B. In particular $\frac{2d}{t_a'' - t_a}$ measures the average ~~velocity~~ speed $\bar{v}$ of the signal in the round-trip journey. Now suppose we could measure the two-way ~~velocity~~ speed $\bar{v}$ without limit. Then we could squeeze the amount $t_a'' - t_a$ to assign a time at A synchronous with t_b at B without limit, so establishing, in the limit, a unique synchrony between A and B. This is the situation in classical physics. But suppose that even in the classical world, there was a (de facto) limit, i.e. a ~~lower bound~~

an observer located at $\bar{r}$, then, even in the classical world, we would be left with an irremediable ambiguity (t_a, t_a'') if the raw corresponded to the emission at reception of the 'first' or 'latest' signal.

Reichenbach now makes a supplementary move. If there is [illegible] way of telling which time instant in the interval (t_a, t_a'') is simultaneous with t_2, then there is no fact-of-the-matter as to this and we are left with a conventional choice of which instant in the interval (t_a, t_a'') to identify as simultaneous with t_2.

If we write

$$t_2 = t_a + \varepsilon (t_a'' - t_a) \qquad (1)$$

where $0 < \varepsilon < 1$

then the conventional ambiguity translates into a choice for ε. In turn this dictates a conventional choice for the one-way speeds of the first signal 3rd

$$\left.\begin{aligned} \overrightarrow{v} &= \frac{d}{t_2 - t_a} = \frac{\bar{v}}{2\varepsilon} \\ \text{while } \overleftarrow{v} &= \frac{d}{t_a'' - t_2} = \frac{\bar{v}}{2(1-\varepsilon)} \end{aligned}\right\} \qquad (2)$$

~~If for the choice $\varepsilon = 1/2$ is $\overrightarrow{v}$~~

So $\overrightarrow{v} = \overleftarrow{v}$ iff $\varepsilon = 1/2$.

This makes the choice $\varepsilon = 1/2$, or any one the simplest, but according to Reichenbach there is nothing 'factual' about such a choice.

Spacetime

9/ it were possible

* Theorem Between two locations at rest (rest) in a given inertial frame I depends only on the distance d between the two locations, as measured in that frame, and on this frame

new para [Suppose, to make contact with SR, the Maximum value for v is the speed ~~velocity~~ of light in vacuo; then the conjugate a'' sections the ostensen of the light-cone constructed at b. The event b is topologically simultaneous with the whole interval (a, a'') instead of a purely one-dimensional simultaneity of the event b with some arbitrarily chosen member of the set (a, a'').

new para [We have talked of a relation of topological simultaneity even in a classical world. But as good a word the two-way speed ~~velocity~~ of the first signal, defined in the sense of relative to k, which it is measured. This of course is the conceptual origin of the Michelson-Morley experiment. ~~But Einstein has mapped a quasi-classical model in which both a two-way velocity which shares with SR the feature that the two-way velocity of the first signal, more independent of inertial frame, than is such a word the topological relation~~ * Inserting ~~depends on~~ the distance between the two events, and ~~should~~ we would have a frame-independent generalization of the classical conception of simultaneity.

new para The situation in SR, it is claimed, is that there is both a first signal, and an invariant two-way speed ~~velocity~~, so the conditions are met, in a perfectly natural way, for introducing the generalized relation of topological simultaneity in the manner described, although the natural distance between

~~Two events as seen from a [illegible] period have, for decades, in general, imputed with [illegible] conventional element, as he shall see a moment.~~

Reichenbach

Grünbaum accepts the conclusion of the Reichenbach arguments about the conventionality of simultaneity in SR, but his reasons are based on ontological rather than epistemological considerations. Adopting a relational theory of time, Grünbaum considers that the ordered [illegible] of the temporal continuum must be based on and understood reduced to relations of causal precedence between events. Grünbaum assumes that a symmetric relation of causal connectibility between events can be established in terms of an antecedently understood notion of a continuous geodesical set of events which comprise the space-time locus of a material particle or (more generally) of a ray of light. Temporal betweenness for events at different spatial locations is dictated by the following definition: [4 5]

Let e_1 and e_2 be events at location A and e_3 an event at location B. Then e_3 is temporally between e_1 and e_2 if the following conditions hold: (We shall write this as $e_3 B e_1 e_2$)

if and only if the following conditions hold:

(1) e_1, e_2, e_3 are members of a possible geodesical set G

(2) If e_3 is deleted from G there is no continuous geodesical subset of G that contains both e_1 and e_2.

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Jean Paul

formally, where R is a variable ranging over all possible

* Proctor The Nelsonian Condition involves two clauses

(1) $\exists G\left(G(e_1, e_2, e_3) \rightarrow \exists G\left(G(e_1, e_3)\right)\right.$

$\left. \wedge \exists G\left(G(e_2, e_3)\right)\right.$

and

(2) $\exists G\left(G(e_1, e_2)\right) \wedge \exists G\left(G(e_2, e_3)\right)$

$\rightarrow \exists G\left(G(e_1, e_2, e_3)\right)$

the first part, being a logical truth, is not controversial. As we shall see later

complete (and hence continuous) Synchronical sets and S as a variable ranging over all possible continuous (but not necessarily complete) Synchronical sets, so $\forall S \exists G (S \subset G)$, then we have

Display

$$e_3 B e_1 e_2 \text{ iff } \exists G \left[G(e_1, e_2, e_3) \wedge \forall S (S \subset G - \{e_3\} \rightarrow \neg S(e_1, e_2)) \right]$$

where $(e_1 \in G) \wedge (e_2 \in G)$ and $(e_3 \in G)$ is written $G(e_1, e_2, e_3)$ and Similarly for $S(e_1, e_2)$.

We notice that the relation B may fail to obtain for two quite different reasons. If Condition (1) holds but not condition (2) in the above definition, then we may say that e_3 is absolutely ~~indeed~~ to be temporal interval (e_1, e_2). But if Condition (1) does not hold then the relation B fails to obtain in a sense that we may want to express by saying that a Between relation is not really very applicable, because e_3 is in an absolute sense Simultaneous with either or both of the events e_1 and e_2. To capture this idea of absolute and temporal simultaneity between an event e_3 of location B and any event, call it e_4, at location A, Einstein makes effective the following definition

DSRCAT

$$e_3 S_G e_4 \text{ iff } \forall G (\neg G(e_3, e_4))$$

We use the notation S_G for this Einsteinian definition of Simultaneity. The relations B and S_G now work in the following way.

No Passing Condition: $\neg \exists G (G(G(e_1, e_2, e_3)) \text{ and } e_1 S_G e_3$ and $e_2 S_G e_3$.

Further second part of the
* It is this Passing Condition which breaks down in

the presence of so-called Fermions (see later).

Now, having explained how our notation, we can formulate the First Signal Principle. Using now e_1, e_2 and e_4 as variables ranging over events at location A, we assert

$$\exists e_1 \exists e_2 [e_3 B e_1, e_2 \wedge \forall e_4 (e_4 B e_1 e_2 \rightarrow e_3 S_G e_4)]$$

Denote

Denoting by $\bar{e}_1$, $\bar{e}_2$ the two events at location A corresponding to the sending and return of the 'First Signal' which arrives at B as event e_3. Then, according to Einstein, e_3 is topologically absolutely simultaneous with any event e_4 at location A which lies temporally between e_1 and e_2. Any particular choice from this range is, for Einstein, a matter of convention: the 'conventionality of Metrical Simultaneity'.

3. Option (ii): The relativity of simultaneity

If we adopt the standard Einstein Synchrony convention of $\varepsilon = 1/2$, ~~then it is easily seen that~~ as two relatively moving inertial frames then it is easy to see that simultaneity becomes relativised to an ~~inter~~ inertial frame. This has nothing to do with relativity theory but ~~one~~ would arise in a Newtonian world if we were sufficiently perverse to use the Einstein convention in such a world. To avoid ~~this~~ complications of calling such a use 'perverse' will be explained in a moment. But to keep things general let us write down more general transformations, which will include the Einstein used

and the Newtonian world as special cases.

Consider an inertial reference frame K' moving with velocity v along the positive X-axis of another inertial frame K. Suppose further that the origins of K and K' coincide for the time $t = 0$. In other words, we restrict ourselves to the simple case of two-dimensional space-time and the most general linear relationship between the coordinates (x', t') of an event relative to K' and the coordinates (x, t) of the same event relative to K must be of the form

$$\left.\begin{aligned} x' &= Ax + Bt \\ t' &= Ct + Dx \end{aligned}\right\} \qquad (3)$$

For $x' = 0$, $x = vt$, so $B = -Av$ and defining $m = -D/C$, we obtain

$$\left.\begin{aligned} x' &= A(x - vt) \\ t' &= C(t - mx) \end{aligned}\right\} \qquad (4)$$

Now we introduce some additional assumptions about the behaviour of moving rods and clocks, viz. a rod moving in K is contracted relative to K by a factor F, while a moving clock is slowed down by the same factor F as judged from K, and further that

$$A = 1/F, \qquad C = 1/G(1 - mv)$$

so we now have

$$\left.\begin{aligned} x' &= 1/F \cdot (x - vt) \\ t' &= 1/G(1 - mv) \cdot (t - mx) \end{aligned}\right\} \qquad (5)$$

In a Newtonian world $F = G = 1$ while in an Einsteinian world $F = G = \sqrt{1 - v^2/c^2}$.

The Galilean transformations correspond to taking $m = 0$ in a Newtonian world.

Shorter

* as viewed from K

But suppose we choose m so as to satisfy the Einstein synchrony convention, that light, assumed to propagate isotropically with speed c relative to K, is also to propagate isotropically (although not with the same magnitude as c) relative to K'. It is very easy to see that this requires $m = v/c^2$, leading to the transformation equations

$$\left.\begin{aligned} x' &= x - vt \\ t' &= \left(1 - v^2/c^2\right)\left(t - \frac{vx}{c^2}\right) \end{aligned}\right\} \quad (6)$$

These equations were first obtained (1977) by Lorentz (1904).

The parameter m, which we may call the synchrony parameter, determines the locus in K of points simultaneous with the origin on K' or the locus

$$t = mx$$

Notice that the X'-axis and T'-axis in K' are Minkowski orthogonal even in the Newtonian case.

The condition for Minkowski orthogonality is simply $m_t m_s = \frac{1}{c^2}$ (-67), where m_t is the slope of a time-like line and m_s the slope of a space-like line. The X'-axis has slope $1/v$ in K while the T'-axis has slope v/c^2, so the condition is satisfied.

Of course the family of Newtonian absolute simultaneity planes are not Minkowski orthogonal since the Minkowski metric is not generally invariant under the transformation.

(6). But the important point to notice is that Einstein synchrony in a moving frame is specified by Reichenbach's ε to the world-line of the moving frame, whatever sort of world we live in, as specified by the physical fourteen Fund G. We shall return to this point in a moment. But first we want to explain why we cannot allow Einstein synchrony as a principled definition in a classical world. In such a world there is in fact no upper limit on speed relative to K and hence no universal cone, synchrony in K is uniquely specified on the topological sense. Now dependence of factual metrical simultaneity must in general allow that K' cannot declare metrically simultaneous events which are not topologically simultaneous relative to K. Hence, in a classical world K' must choose $m = 0$, the choice $m = v/c^2$ to centuries the above constraint on the relationship between topological simultaneity relative to K and metrical simultaneity relative to K'. But in the Einstein world, with a finite upper limit to speed, provided by the propagation of light in vacuo, this constraint does not rule out the Einstein convention for any velocity v up to the maximum c.

In other words, in SR the topological light-cone specified by the boundaries of the future and past light cone with slopes $\pm 1/c$ exactly permit the full range of Einstein synchrony in many frames defined

by Hopes $\frac{v}{c^2}$ with $|v| < c$.
To put the point a little differently, non-standard synchrony in K matches up in a one-one relationship with standard synchrony in an appropriately moving frame — we just have to choose the relevant frame so that $v/c^2 = m$, where m is the remaining parameter lying in the interval $(-1/c, +1/c)$, specifying the non-standard synchrony in K.

new para

So the proponents of Option (2) cannot be ruled out of court on the Fourteenth used in the way we used for the Newtonian word.

new para

So much as agreed between the proponents of Option (1) and Option (2).
But Option (2) now deploys a number of arguments to show that the Fourteen convention in all spatial frames is not just a permissible conventional choice, but is actually the fact-of-the-matter.

I. (first of all) there are arguments (purporting) to show that non-standard synchronies are unsuitable candidates for defining simultaneity because they do not (specify) an equivalence relation on events. If we chose a fixed Reichenbach ε-parameter to specify non-standard (simultaneity) then it is easy to show (that) simultaneity so defined is neither symmetric nor transitive.

Do not understand.

To see how this comes about, we first note the relation between the parameters ε and m for two-dimensional spacetime. It is easily checked that, in the frame K,

for $x > 0$

$$\xi = \frac{1}{2}\frac{(1-mc)}{(1-mv)}(1+v/c)\lambda \qquad \varepsilon = \tfrac{1}{2}(1-mc)$$

while for $x < 0$

$$\xi = \frac{1}{2}\frac{(1+mc)(1-v/c)}{1-mv} \qquad \varepsilon = \tfrac{1}{2}(1+mc)$$

So if ε has a fixed value, we must replace m by $-m'$ for the negative part of the line of simultaneity. So, for fixed ε the line of gradient of fixed slope m becomes replaced by a pair of lines of slopes $\pm m$. only for the case $m=0$ which defines standard synchrony in K does a fixed ε, viz. $\tfrac{1}{2}$, correlate to a single line.

These results are readily generalized to the Relativistic four-dimensional spacetime. For fixed ε, the surface of simultaneity as a hyperplane which degenerates into a plane when $\varepsilon = \tfrac{1}{2}$.

But viewing when the surface of simultaneity as a hyperplane that we can recover symmetry and transitivity and hence an equivalence relation.

But the remedy is quite simple - choose ε to be an arbitrary function of direction so as to render the hyperplane. The details are just as the results already given in Eq. (1). Then not standard synchrony just corresponds to hyperplanes of simultaneity that are not orthogonal to the time-axis of the inertial frame.

At one time there was a great deal of discussion in the literature about the resolution of non standard synchrony. But this we submit has no ~~need~~ remedial relevance to the discussion of the results of non standard synchrony.

II. An important consideration is the fact that slow-clock-transport gives to such out standard synchrony in an Einsteinian World.

Of course, in general, in SR if two standard clocks are synchronized at the same location and then moved along different spacetime paths and reunited, they will no longer be in synchrony — that is the origin of the twin paradox, which we will discuss in more detail in a moment. But Bridgman was the first to suggest that if ~~the~~ a clocks was ~~were moved more and more slowly as~~ moved the infinitely slowly from location A to location B, fixed locations related to a frame K, then the clock, or ensemble of B would come in reading with standard synchrony defined relative to K. We must first of all point out that we mean by 'infinitely slowly' that the clock did not move at all it would never reach B! So we must consider a limiting procedure for a sequence of clocks moving more and more slowly from A to B. But how do we define the speed of velocity, the

Reichenbach & Ulbeck doesn't prove a synchrony convention? To most ostensibly procedure would seem to be specify the velocities using an arbitrary E-parameter and then show that ~~the discrepancy with standard synchrony~~ { we take the double limit, first letting $\to 0$ velocities tend to zero, and then letting $\varepsilon \to \frac{1}{2}$, the discrepancy between the time at B recorded by the moving clock and the time at B specified by E-synchrony itself tends to zero. #11

There are two reasons that we can make to this admitted philosophical fact about Low-clock-transport in SR. Firstly, we can follow Grünbaum #12 who identifies a tangled convention in the assumption that the intervals Low clock measures the same duration when transported between events A and events at B as does the clock remaining at A and in standard synchrony with clocks at B. Grünbaum's qualms in this area are for his general arguments for the metrical amorphousness of a linear continuum. But we do not have to resort to Grünbaum's (intervals) Cartesian stories #13 about metrical amorphousness to reject the Low-clock-transport argument for the facticity of standard synchrony. We could extend argue as follows. There is no absolute fact of the matter in the term 'Low'. When we (estimate) Low clocks moving from K, then Low-clock-transport will pick out standard synchrony in K,

but this, as we have already noted, corresponds to mainstream practice in K.

new para

On brief, if you are already committed to option (2) as that annotation is returned to a home, then flow-back transport can quickly feed out the Eustace conviction as the court may be inclined to interpret the statutory order. But the defender's option (1) will see nothing in the argument to show that annotations duly to factually returned to an vested rights, and will conclude that the choice between it's flow-back transport and K-11's flow-back transport is indeed a conventional one. Indeed he will go on to claim that the conventional interpretation permitted by the statute of flow-back transport entered under regard to all foreign cases would correspond to the intended & typical investment & method of export conviction by Parliament and guidance. So arising against the court's adoption of the pursuer's argument as to find the conventionality of injunctions using the method of pursuer's the option (2) proponent against themselves.

III. The postulate of Plato, the foci of standard hyperbolas, have made great play with the fact that standard hyperbolas, unlike non-standard hyperbolas is easily definable. 13 14
The basic idea is very simple. We have already noted that standard hyperbolas in a frame K' coincide in Minkowski orthogonality between the line of simultaneity, i.e. the x'-axis and the world-line of the origin of the frame, i.e. the T'-axis.

new para
But Minkowski orthogonality is easily shown to be definable in causal terms by the following construction, due originally to Robb. We first note ~~the following~~ a simple lemma of plane Euclidean coordinate geometry.

Lemma

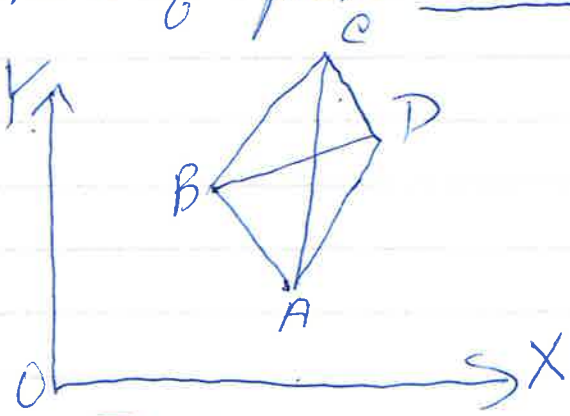


Fig. 2.

Lemma If ABCD is a parallelogram whose adjacent sides AD, AB have slopes $(+m)$ and $(-m)$, then if the diagonals AC and BD have slopes m_1 and m_2 respectively, the quantities m, m_1 and m_2 satisfy the relation $m_1 m_2 = m^2$.

Now apply this result to two-dimensional spacetime (i.e. replace the Y-axis with a T-axis) and let AB, AD, BC and DC be the spacetime loci ~~lines~~

of rays of light, so $m = 1/c$, then the
Construction shows that for any timelike
line AC of slope m, and spacelike line BD
of slope m_2, we have $m m_2 = 1/c^2$,
but this is just the Lorentz condition for
Minkowski orthogonality.
Below we have demonstrated the construction
for two-dimensional spacetime it generalizes
immediately to the four-dimensional case.
Robert (1977) has proved the remarkable
result that standard synchrony is the only non-trivial
two-place relation of simultaneity defendable from
the relation of causal connectibility and
the world-line of the origin of an
inertial frame of reference.
But there are two arguments we could
use against the significance of Robert's
result. Firstly, we can dispute that simultaneity
need be an equivalence relation, what
looks bad in the direction of time (1) and
the clearly causally defendable notion of temporal
simultaneity. But secondly we can use the
same argument as we did for Reichenbach-
Grünbaum to demonstrate that we are
faced with a countless class of
standard synchronies defined à la Robert
for all possible inertial frames, at the
latitude, as we have seen, fails as bad
to sort the true latitude, as we conventionally
by topological considerations. The use of
argument has been particularly urged by
Davies (1983) in his discussion of the
significance of Robert's result.

~~Before turning to Option (3), we want to suggest for a moment to evaluate Einstein's claim, that if trains were known to exist they would have to accept Stoday as being forced on them conventionally. revert to a Lorentz simultaneity relation.~~

4. Option (3): The Neo-Lorentzian

There has been a steady stream of literature over the years promoting a neo-Lorentzian understanding of SR. [14][15] There is a privileged 'aether frame' with respect to which the propagation of light is really isotropic. The apparent isotropic propagation of light, and the invariance of the two-way speed of light (in moving frames) is due to the fact that moving rods and clocks are affected according to the Lorentz contraction and dilatation factors. So Nature conspires to hide from us the empirical detection of the aether frame. But the neo-Lorentzians argue that only a crude positivism would lead us to believe that because we cannot detect the aether frame, that such a frame does not exist. They point to the explanatory power of the aether frame hypothesis in role embedding explaining the contraction and dilatation effects which Einstein, we may claim, just assumes rather than explains. Accepting the neo-Lorentzian model depends on one's understanding of what constitutes a scientific explanation. To

Heisenberg seem to be arguing on the basis of a crude causal model which has been used to justify the existence of a hidden ether. Ironically enough, SR relativistic ideas about the possible existence of tachyons travel faster than light. If tachyons exist then Stückelberg himself has argued that they could be used to pick out a unique, preferred relativistic frame. This argument has been answered critically in Redhead (1983). Since this whole question is of considerable conceptual importance in understanding the significance of SR, we shall pause to summarize the discussion here.

He thought that SR actually licenses the possibility of superluminal particles arises from the view that SR is committed only

to an invariance principle, viz. the invariance of the four-dimensional Minkowski interval, which implies in particular the invariance of the light speed in velocity space. Lorentz transformations allow any point inside the light sphere to be transformed to any other interior point, the corresponding motions of the exterior points corresponding indeed to Lobachevsky's hyperbolic geometry. The region exterior to the light sphere in velocity space is also invariant under Lorentz transformations. The significance of these mathematical facts about the effect of Lorentz transformations on points in velocity space is that the light sphere is an impenetrable barrier: a particle with speed less than that of light can never be Lorentz-boosted across the light sphere. This is the sense in which SR incorporates a First Signal Principle: no material particle, or more generally, localized energy packet can move faster than light. But suppose a particle, a so-called tachyon, moved always faster than light. Lorentz transformations could never slow it down so that it moved to the interior of the light sphere, which appears formally as an impenetrable barrier, but now seems just to slow, rather than slow to fast. It is easy to show that if tachyons move at any speed greater than that of light then an appropriate Lorentz boost can generate any negative velocity however great, i.e. there is no upper limit to the speed of a tachyon.

* in the a moving frame K'

Sciama now argues that if there were tachyons and hence infinite velocity tachyons (speaking loosely) then we could do in the same situation as in the Newtonian world, these infinite velocity tachyons would push out a unique hyperplane, by the usual causal argument and the conventional Lorentz topological simultaneity would be dismissed.

✓ new para

It is unfounded to consider just the case of acoustic velocity. Consider a. round wave wfront propagating with velocity w along the positive and negative X-axis relative to a frame K, where $w > c$. [~~If we synchronize clocks to obtain~~ To fix an ~~hyperplane of event clocks to~~] does define the hyperplane of the frame & of the event wave relative to K (assume for simplicity two-dimensional spacetime) as of the form

$$\frac{1}{w^2}\frac{\partial^2\phi}{\partial t^2} - \frac{\partial^2\phi}{\partial x^2} = 0 \qquad (7)$$

Then this hyperbolic pde could be used to identify a latitude on topological simultaneity at a distance x of $\pm x/w$ about the event determined in retarded coordinates at the location by working the acoustic synchronization another of ~~extreme~~ sound propagators on both directions along the X-axis. But now suppose we introduce the Einstein ~~Stat~~ synchronization in ~~for~~ a ~~increase~~ [analogue] to Cetov well now contravene the ~~also~~ logically specific acceptable latitude when $\sqrt{x}/c^2 > x/w$ i.e. $v > \frac{c}{\sqrt{x}}$. C. Or $w \to \infty$, ~~the~~ the Einstein convention will do

ontologically disallowed for any $v > 0$, i.e. the theory decides is forced to use the hyperbolic one, which is now defined uniquely in K.

In order to obtain the transformation equations between K and K' one easily shows to reduce to the Lorentz form[2021]

$$\left.\begin{aligned} x' &= \frac{1}{\sqrt{1-v^2/c^2}}(x - vt) \\ t' &= \sqrt{1-v^2/c^2}\cdot t \end{aligned}\right\} \qquad (8)$$

Einstein, together with Sjödin and Salmon[21,22] ~~after~~ efforts aimed ~~that~~ this is the analysis ~~that~~ ~~applies~~ on the ~~presence~~ presence of tachyons. But ~~things~~ ~~are not clear~~ the situation is not that simple. Tachyons ~~they~~ t are from consideration of a Lorentz-invariant equation ~~quite different~~ typically the Klein-Gordon equation with an imaginary mass-parameter, viz.

$$\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} - \frac{\partial^2\phi}{\partial x^2} - \mu^2\phi = 0. \qquad (9)$$

The solutions of this equation have quite different properties from the Lorentz-invariant equation (7). Non-projected to quantization (9) describes tachyons moving with any velocity between c and ∞. Eq. (7) by contrast describes the propagation of sound waves with a unique velocity $v < c$. Eq. (9) also accommodates tachyons propagating backwards in time, with velocities between c and ∞, but does turn out to have rejected aspects

and a Reconfiguration Principle is standardly invoked in which tachyons formally directed as propagating backwards in time are reinterpreted as antitachyons traveling forwards in time with positive energies. The notion of emitting or absorbing a tachyon is not an absolute one. The emission of a tachyon in one frame may identified with the absorption of an antitachyon in other frames and vice versa. So the possibility of generating a causal loop paradox and a chronology violation might sustain a symmetric relation of causal connectibility between events at space-like separation, although an asymmetric cause-effect relation could not be established. Here it is in terms of causal connectibility that principles grounds the notion of temporal betweenness, we shall discuss in a moment how this explains the hypothesized presence of tachyons. But first we want to explain the [illegible] in very tachyons to establish absolute synchrony.

New paragraph

Suppose an observer stationary in [illegible] frame K attempts to establish absolute synchrony using an infinite-velocity tachyon (usually referred to as the hypothetical instantaneous tachyon), We suppose that this instantaneous tachyon and

* Inserter (see Fig. 4 (iii)).

identified as frame-content relative to the coordinates of the Poincaré conventions in the K frame. In Fig. 3 we illustrate a to-and-fro signalling between A and B using 'locally transcendent' tachyons, where the clock e_4 at A, which is retrocedly synchronized with e_3 at B is squeezed between the emission (e_1) and return (e_2) of the tachyon signal.

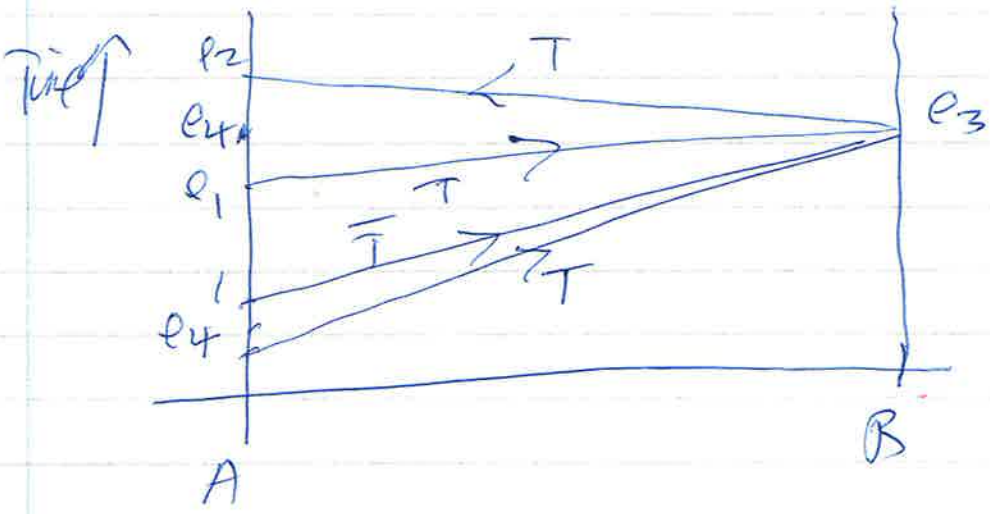


Fig. 3. Tachyon synchronization as used from K ($\bar{T}$ = tachyon, T = antitachyon)

But now consider the description of what is seen on from the point of view of an observer attached to the frame K' moving with uniform velocity v relative to K, who also accepts the Einstein convention. I will deny that the physical processes being described relative to K establish synchronicity between e_3 and e_4 at all. What is happening as seen from K' is that a tachyon-antitachyon pair is emitted at e_3, the tachyon arriving at e_2 and the antitachyon at e_1.

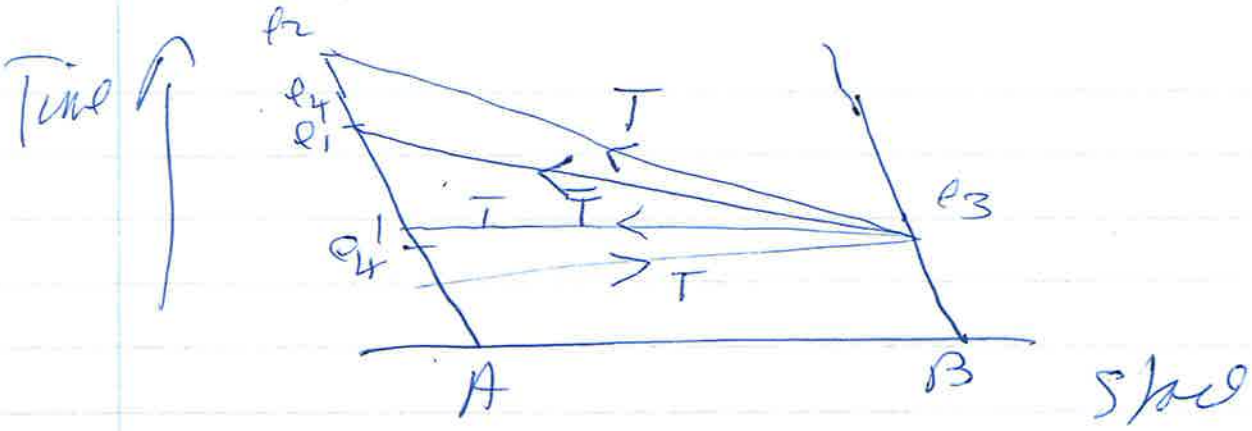


Fig. 4. Tachyon synchronization as viewed from K'

The reason why from the point of view of K', no simultaneity relation is being established between e_3 and e_4 is that [illegible] of events e_1, e_3 or is not a coincidence, since two particles not at rest, are clearly involved. Of course relative to K, there is another physical process, which he/she the observer attached to K, denotes as identifying e_3 or simultaneous with an event e_4, squeezed between tachyons travelling from A to B and back again with nearly transmitted velocities as viewed from K'. But the observer attached to K will insist the claim that e_3 and e_4 are truly simultaneous by pointing out that what is going on is a tachyon-anti-tachyon pair annihilation at e_3 as illustrated on Fig. 3. Notice the asymmetry in the situation - the observer in K cannot describe the simultaneity claim by the observer in K', since he is going to invoke the Reinterpretation Principle to circumvent the K' observer's claim that ...

e_3 and e_4 are coordinates. In fact both observers must allow that what counts as a coincidence for e_1 to e_3 and e_2 to e_4 is a matter that depends on the observer and has no absolute significance. It is only with relativization of relations signalling that an unambiguous answer to coincidence for the 2-and-2 case is available.

The difficulty with the Grünbaum relation of simultaneity the relation we characterized as S_G is now apparent. The betweenness relation B and the simultaneity relation S_G no longer satisfy the Matching Condition (see p. above) Viii. More specifically, it is the second part of the Matching Condition that fails when tachyon interactions are taken into account. In Redhead (1983) it was argued ~~claimed~~ that the Matching Condition was a necessary ~~not to be denied~~ requirement in capturing the notion of simultaneity. A suggested revised definition of simultaneity, S_R, was stated as follows.

Let e_5 be a variable ranging over events at location 4)

Then $e_3 S_R e_4$ iff $\forall e_5 \forall G(\neg G(e_3, e_4, e_5))$

Clearly $e_3 S_G e_4 \rightarrow e_3 S_R e_4$ but not conversely. S_R will now satisfy the Matching Condition provided we allow the logical validity

$\exists e_5 \exists G (G(e_1, e_3, e_5) \wedge \exists\, !e_5 \exists G (G(e_2, e_3, e_5)) \rightarrow \exists G (G(e_1, e_2, e_3))$

This condition is perfectly acceptable for interactions mediated by tachyons, photons or 'subluminal' particles (often referred to in the literature as bradyons).

New Ford

With the revised definition, the range of events at location A, that are simultaneous with event E_3 at location B is exactly the same as with photon signalling. All this is just the right latitude to justify us in employing the Einstein convention in any (subluminal) frame K, and so underpins ontologically speaking, the usual Lorentz transformations and the associated invariance principle of the Newtonian metric. But note that we are still able to allow two events to causally connect via tachyon interaction. Even at Hyde-like separation intervals we distinguish Causal connectibility from the relativistic simultaneity relation as the case of logic stated. In Bedford (1983) these ideas were applied about tentatively, to recently particularly in quantum mechanics and the constraints of ~~the~~ SR.

New Ford

Following Bedford (1983) let us then again use the key distinction between bradyons and our hypothetical superluminal photons (acoustic particles). With 'infinite velocity' photons we can establish absolute synchrony. It is true of course, that relative to some other frame K' (which employs the Einstein synchronization convention) the photons will no longer appear to have infinite velocity, but because the photon experiences no 'Lorentz-invariant',

non-transcendent phenomena cannot also occur as possible physical processes relative to the K-frame. If we then take possibility that goes over to the physical situation as holding K and K' as the tachyon case when disagreement arose as to whether e_3 was simultaneous with e_4 or e_4'. To put it another way, transcendent phenomena pick out uniquely an absolute frame of reference relative to which they all have some (infinite) velocity in all spatial directions. Tachyons because they satisfy Lorentz-invariant equations, fail to pick out a preferred frame.

5. Conclusion

It is often argued that adoption of the unobserved theories of the conventionalist of natural geometry depends on adopting a reductionist causal theory of time, and it is rightly contended that notions such as continuous geodesical path of events seem to be grounded through our notions of temporal order, while furthermore the whole question of geometries, hypotheses, etc., becomes quite distant at the quantum-mechanical level. However, our discussion has shown that often (1) can be regarded as standing on its own feet as a generalization of classical simultaneity. Although we give up properties, we crucially retain the feature of invariance, that topological equivalence is a relation

* Inspection

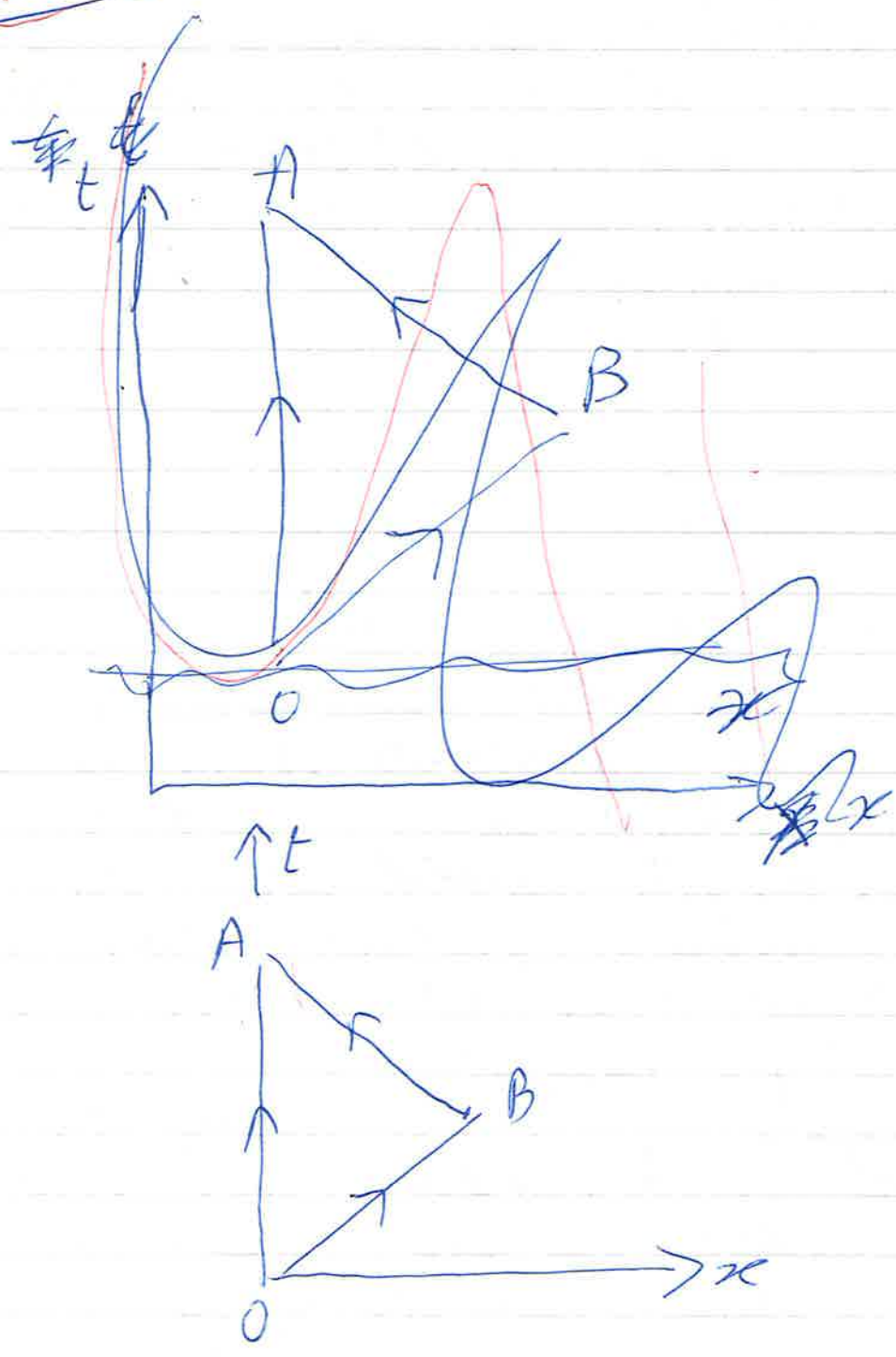


Fig. 5.

that is agreed on by all observers, just as in the case of the classical concept. The terminology 'relativity' for Einstein's theory is deeply very misleading. The original predictions are still backed as experiments, but the invariants are different from those arising in classical physics. Like all of the Lorentz transformations of the invariant group, it is only to be expected that the different observers will not. All the factual experimental evidence can be confirmed in terms of invariants, i.e. in a coordinate-free formulation.

New para

As an example of the practical difficulties afforded by option (1) we consider briefly the ~~twin paradox~~ celebrated twin paradox. Relative to a frame K, twin ~~A~~ 2 remains at rest, while twin B at location A while twin 3 moves to location B' relative to K, with velocity v to ~~location A~~ Einstein LEAVE SPACE HERE FOR Fig. 5. Symbols the two twins coincide at special location O. Then 2 remains at rest relative to a frame K while twin 3 moves to special location B with velocity v relative to standard synchrony in frame K, and then returns with velocity $-v$ to meet 2 again at special location A. If 2T is the time lapse shown by a standard clock fixed to 2, then energy is

Fresh paragraph

Fresh Reb

~~proper~~ time lapse measured by a clock attached to β is $2T\sqrt{1-v^2/c^2}$. This gives very simply, just what the clocks measure the Lorentzschi interval, & call it s, and relative to K, we calculate s' and.

$$s(OA) = 2T$$
$$s(OB) = s(BA) = T\sqrt{1-v^2/c^2}.$$ } Display.

$$\text{So } s(OBA) = 2T\sqrt{1-v^2/c^2}$$

Again relative to K, the lines of simultaneity are ~~retogrid~~ to OA and of the ~~past~~ proper time $\tau_\beta(P)$ along β's trajectory up to a point ~~delayed~~ by K to be simultaneous with P on α's trajectory, against the proper time $\tau_\alpha(P)$ measured up to P along α's trajectory we obtain the following simple graph. (Fig. 6.)

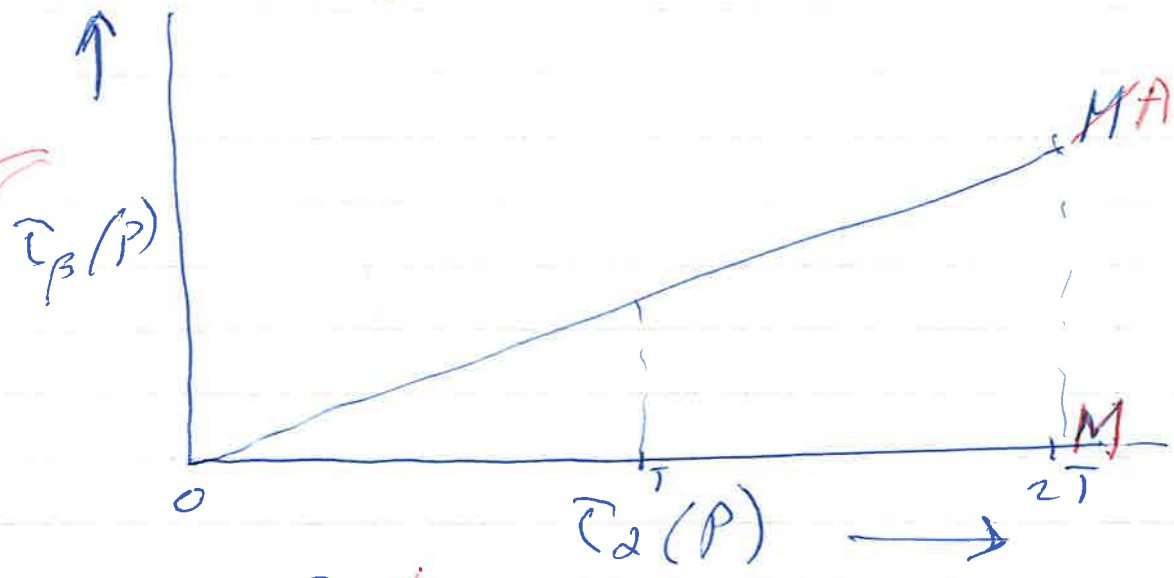


Fig. 6.

— a straight line ~~OMOA~~ where, $OM = 2T$ and $MA = 2T\sqrt{1-v^2/c^2}$.

Space here is used for Fig. 6.

This much is agreed by everyone. The arguments all start with explaining

these results from the first β's point of view.

But from the ρ approaching the problem using option (1) it is all very simple.

For each value of $\tau_2(P)$ there is a latitude in possible values for $\tau_3(P)$ depending on which point on β's trajectory is taken to be retrically simultaneous with point P on δ's trajectory. So there will be an upper and lower limit $\tau^u_\beta(P)$ and $\tau^l_\beta(P)$ for each value of $\tau_2(P)$ dictated by the constraints of topological simultaneity. It remains to calculate the functions $\tau^u_\beta(P)$ and $\tau^l_\beta(P)$, considered as functions of $\tau_2(P)$.

Imagine the point P to move from O along the worldline of δ, i.e. along OA in Fig. 7, and consider the intersection of the light cone at P in various positions with the world-line of β, viz. OBA.

For $0 \le \tau_2(P) \le T(1-v/c)$ the intersection of both forward and backward light cone is on the segment OB. For $T(1-v/c) \le \tau_2(P) \le T(1+v/c)$ the backward light cone intersects OB and the forward light cone BA. For $T(1+v/c) \le \tau_2(P) \le 2T$ both intersections are with the segment BA' (see Fig. 7 cases (i), (ii) and (iii).)

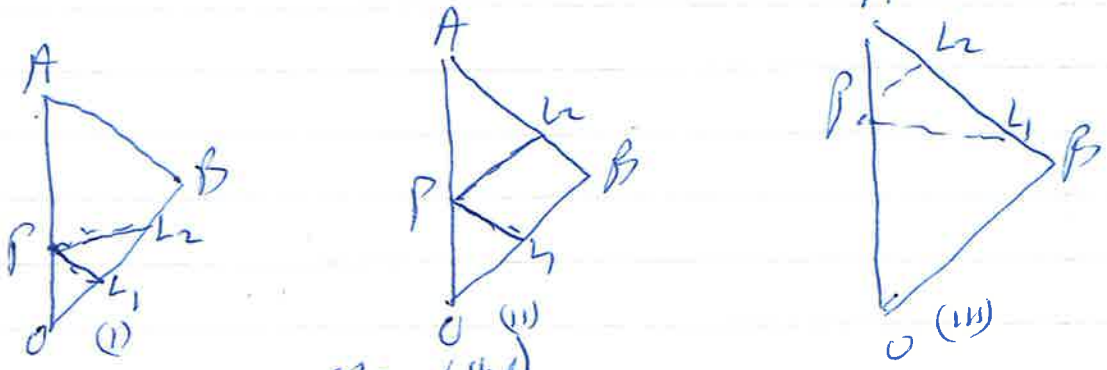


Fig. 7 Light lines (straight dotted) joining P to OBA

~~Then $\vec{\tau}_\beta^{\,u}(P) = S(OL_2)$ and $\vec{\tau}_\beta^{\,e}$~~

Now $\vec{\tau}_\beta^{\,e}(P) = \vec{S}(OL_1)$ and $\vec{\tau}_\beta^{\,u}(P) = \vec{S}(OL_2)$ the interval being computed along the trajectory OSA.

The calculations are straightforward and produce the diagram (fig. 8.)

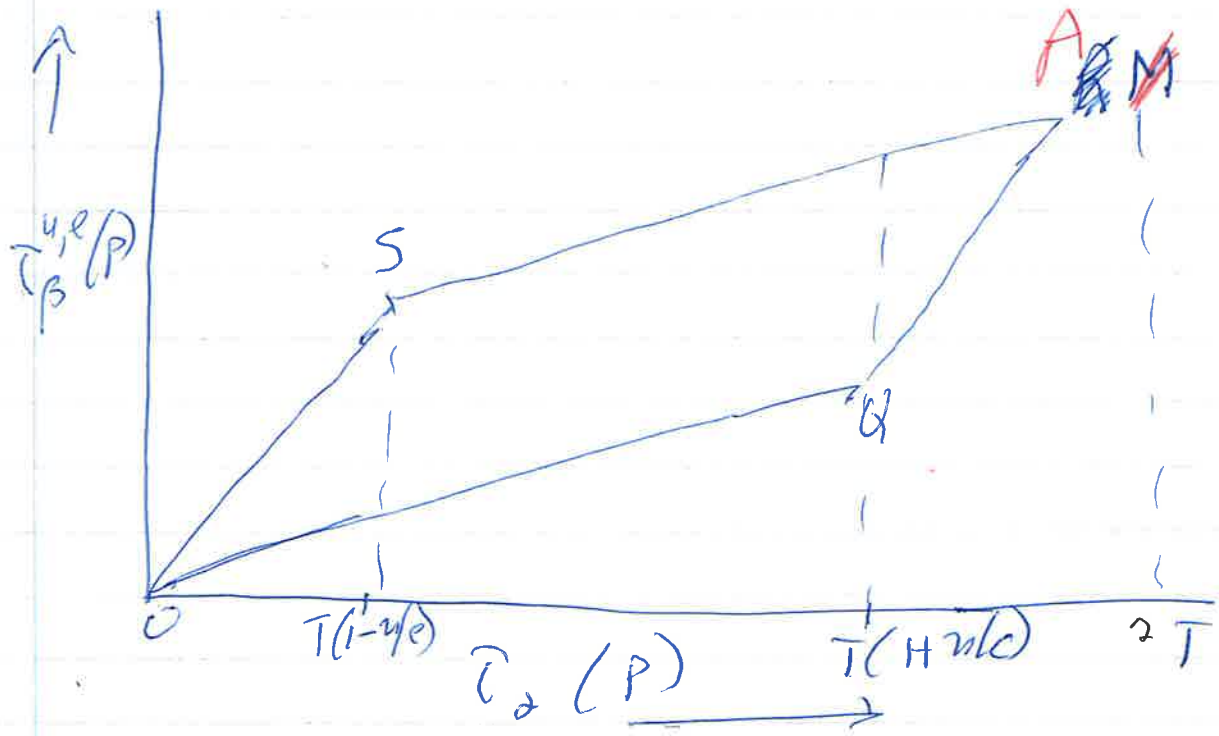


Fig. 8.

$\tau_\beta^{\,u}(P)$ is given by the sides OS SA of the parallelogram OSAS'.

The slope of OS is $\sqrt{\frac{1+u/c}{1-u/c}}$ and the slope of SA is $\sqrt{\frac{1-u/c}{1+u/c}}$, the abscissa of S being $T(1-u/c)$ and of A, $T(1+u/c)$.

$\tau_\beta^{\,e}(P)$ is given by the sides OQ, QA of the parallelogram.

The line OA of Fig. 6. or simply the diagonal of the parallelogram OQAS in Fig. 8.

~~inverted~~ * and lying within the parallelogram ~~OAMS~~ OBAS

But exploiting the conventionality of simultaneity means that any non-decreasing curve joining O and A is an acceptable 'story' of how the clock readings are related at various readings of S's clock

rewrite Let us take two examples that have been discussed in the literature.

(1) Take the average of $\tau_\beta^+(\beta)$ and $\tau_\beta^-(\beta)$ up to $\tau_2(\beta) = T(1-u/c)$ and extrapolate to $\tau_2(\beta) = T(1-u^2/c^2)$. Do the same for $\tau_2(\beta)$ beyond $T(1+u/c)$ and extrapolate back to $T(1+u^2/c^2)$. Complete with horizontal section between $\tau_2(\beta) = T(1-u^2/c^2)$ and $\tau_2(\beta) = T(1+u^2/c^2)$

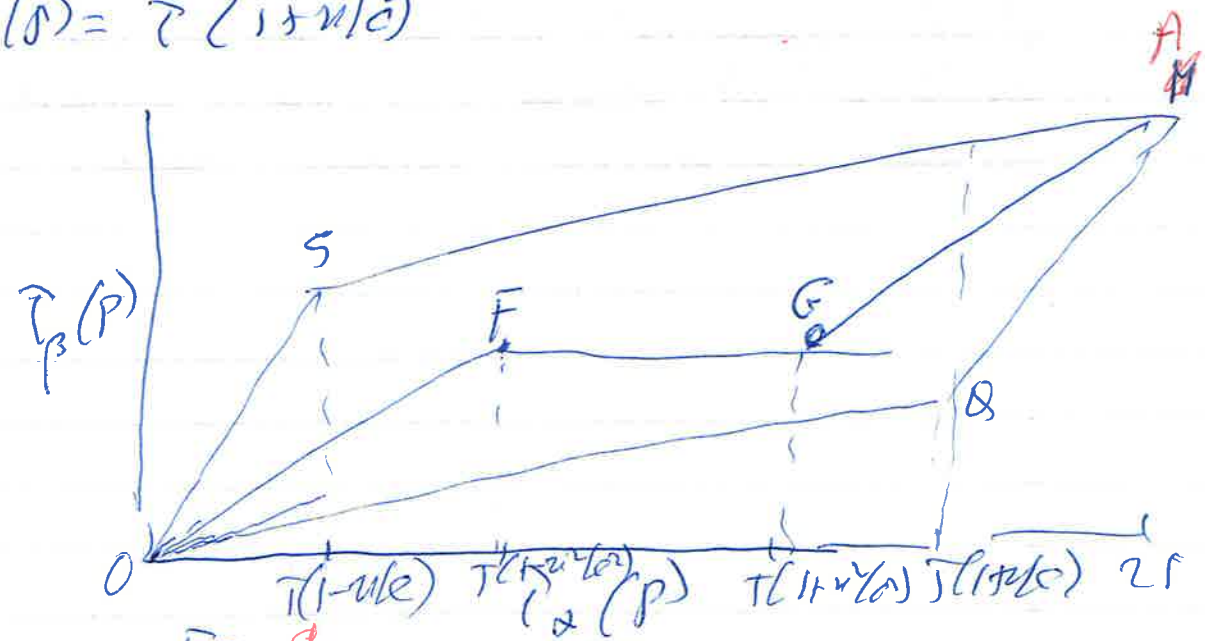


Fig. 9 -

The resulting curve is OFGMA where the slope of OF and GM are $\frac{1}{\sqrt{1-u^2/c^2}}$ (see Fig. 9)

This shows S's clock running ahead of S's along the sections OF and GA, but systematically standing still along FG, so allowance for the differential agein of S to take place!

Newkars
Frontier

Assumes of the two postulates in the
Context of Special relativity typically
adopt the analogues of equation (1)
above but with more complicated
descriptions of the accelerations involved in particular β's
velocity. In the limit of infinite
acceleration the graph of Fig 9.
is then assumed to be bounded
between FG being then asymptotic to
the (limit) theory axes of β's and
by the postulates maintained
field Sparked by the Equivalence
Principle [25] But once the Doctrine is
that introduces Special relativity is
rejected and theory the calculations
of the relevant proper times for clocks
attached to A and B being independent
of the coordinate system used.

(2) Take the average of $\tau_\beta^A(P)$ and $\tau_\beta^B(P)$ everywhere. This is illustrated in Fig. 10.

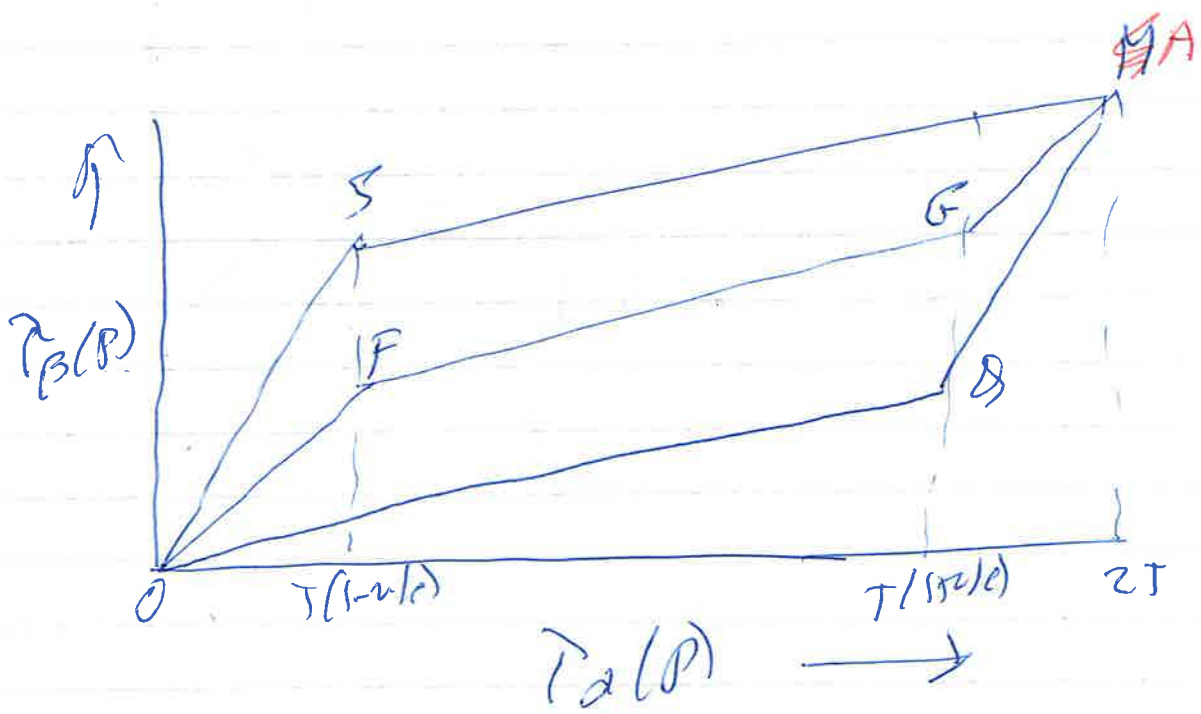
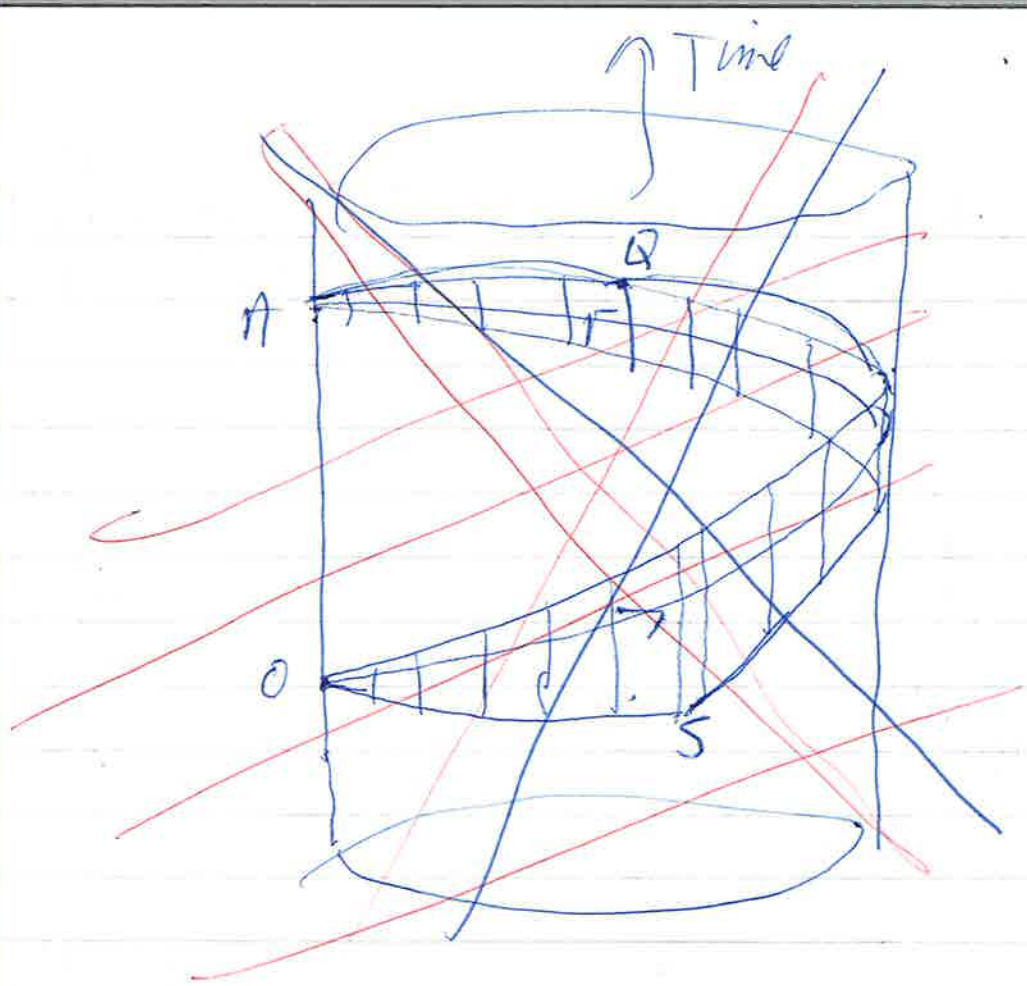


Fig. 10.

Along OF β is gaining (at rate of $1/\sqrt{1-v^2/c^2}$), similarly along GA.
But along FG β is losing (at rate of FG is just $\sqrt{\frac{1-v/c}{1+v/c}}$.)

A noteworthy feature of this solution is that β's estimate of α's distance $\frac{1}{2c}\left(\tau_\beta^A(P) - \tau_\beta^B(P)\right)$ remains constant between $\tau_\alpha(P) = T(1-v/c)$ and $\tau_\alpha(P) = T(1+v/c)$, a curious regularity first noticed by Einstein but indeed is traceable down from the geometry of Fig. 10.

New para The above analysis can also be applied directly to the twin paradox in Special Relativity. The twin β travels along OA while β's twin α remains at rest undergoing velocity along DBA



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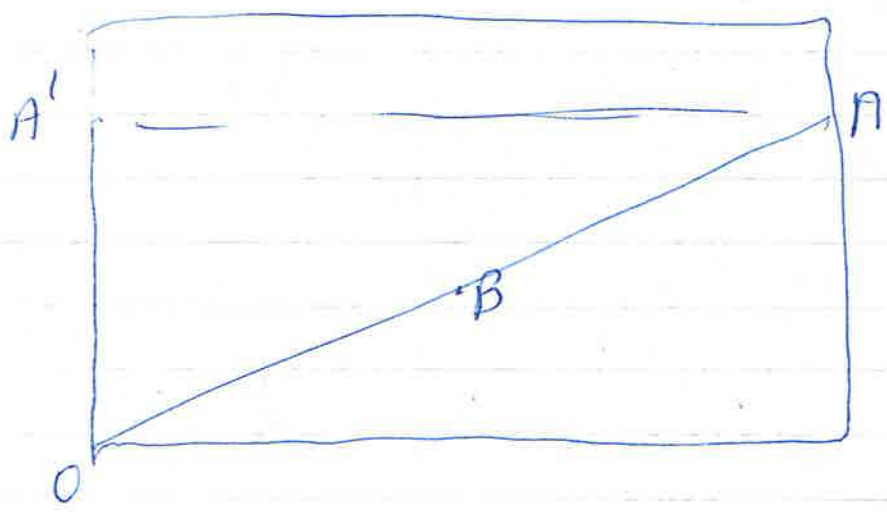

Fig. 12.

To meet 2 again at A (see Fig. 11.)

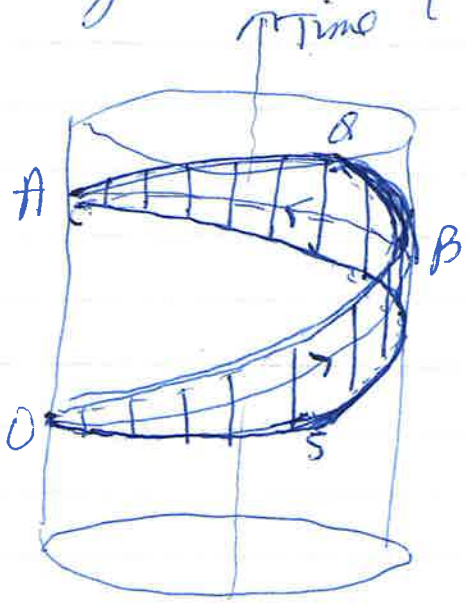

Fig. 11.

To find the Calculate on proper time of B that may be judged topologically simultaneous with a given lapse of proper time of A, we may have to take account of light propagation in both a clockwise and anticlockwise sense.
(Unwrapping the surface of the cylinder as shown Fig. 12. where the points A and A' are to be identified:

Section

SPACE HERE FOR Fig 12

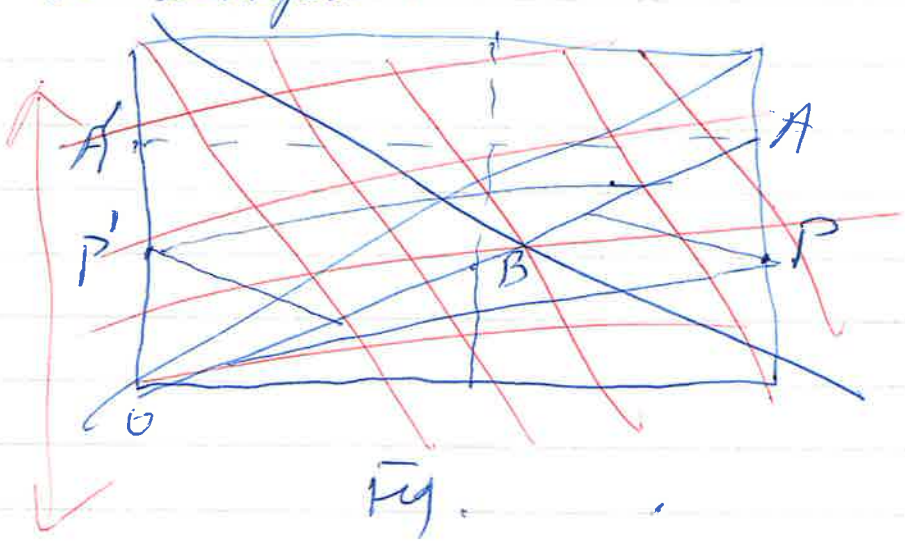

Fig.

Folding about the vertical line through B brings us back to Fig. 5., with AS A' again

* Spectrum

~~Therefore~~ It is easily checked that all constraints of distance and orientation kept propagation on topological simultaneity exactly corresponds to the situation sketched in Fig. 7. (I), (II) and (III).

So we never again a parallelogram like ○ABAB in Fig. 8.

For fixed $\tau_B(P)$ this diagram restricts the range of $\tau_A(P)$ that can be judged topological simultaneous and this variable range can be represented, as in Fig. 11. by wrapping the parallelogram back around the cylinder as indicated by the ~~dotted lines~~ vertical hatching a region shaded vertically.

Any more detailed story about reading on B so to be related by what clock reading on A is again a matter of convention.

So often (I) gives a very satisfactory account of these matters.

New para But what about the A-theorist and the role of the moving present? This is quite definitely ruled out on the basis of option (I); since the representation of topological simultaneity carried with the transients of co-reality would imply that all events are co-real.

Next ~~This is the real significance of the~~ Reichenbach - Putnam arguments (discussion of this ~~point~~ formerly) ~~they~~ to show the incompatibility of option (I) with the A-theory of time. The only way to rescue an A-theory is to go for option (2) and relativize co-reality to a frame, or any even a very small option, or to use option (3) to pick out

a privileged frame. And the answer
come down firmly in favour of (1),
our own feeling is that SR is
inconsistent with the A-theory of
time.

Finally what about general relativity (GR)?
Many of the specific arguments that
work in SR fail in GR. For example,
the contrast structure no longer (modulo deformations)
the metric, we need to know how to
prescribe, structure like the notion of parallel
a global time (at all) depends on the peculiar
class of peculiar manifolds
they belong to, which may not, in general, be
possible. But locally GR has the
structure of SR (in the neighborhood
that the tangent leads to the
manifold and the structure of Newtonian
spacetime). So the analysis we have
given for SR (for local analysis
of simultaneity) works in GR.
It is in the global aspects of the
concept of simultaneity that GR
introduces new conceptual puzzles
that lie beyond the horizon of ~~the~~
~~paper~~ the present work.

Footnotes

1. In this paper we shall not be concerned with the important question of whether this relation is itself induced by a primary relation that obtains between spacetime points, ~~at which the events happen~~ interpreted in some sense 'substantivally'. For a recent comprehensive discussion see Earman (1989).

2. Better terminology might be 'displacement tensor', and the relation is not a general topological invariant of Riemannian space. But we shall in what follows (contrary to usage) use the well entrenched Backhaus Grünbaum locution in the present paper.

3. See Reichenbach (1957) and Grünbaum (1973). The ~~canon~~ arguments (of course) really go back to Poincaré (1905) who declares the defendant status of his own choice of synchrony.

4. See page 42.

5. ~~4.~~ We follow here the reconstruction of Grünbaum's ideas provided in Redhead (1983).

6. ~~5.~~ The fact that the relationship is a linear one can be justified on the basis of the homogeneity of space and time. See for example Podlaha and Navratil (1966). The following treatment of spacetime transformations follows in Redhead (1983). See also Podlaha and Navratil (1967) and Sjödin (1979).

7. ~~6.~~ Choosing units so that $c = 1$, gives the coefficients of Reichenbach's epsilons ~~$\varepsilon_1 \varepsilon_2 = -1$~~ on our two-dimensional worlds as $\varepsilon_1 \varepsilon_2 = -1$, as contrasted with the usual Cartesian condition $\varepsilon_1 \varepsilon_2 = +1$.

7. See Tonetti (1983), pp 225-226 & Friedman (1983) pp. 171-172.

8. cp (for example) Grünbaum (1962), Quinn (1974), Roth Singh (1975).

9. See Bridgman (1962) pp. 64-67.

10. cp Friedman (1977).

11. See Grünbaum (1969a).

12. For discussion of Grünbaum's theses, see in particular Tonetti (1983) and references detailed therein, also Friedman (1983) and Grünbaum's own reply to his critics in Grünbaum (1969a).

13. See Robb (1914), Robb (1921) Robb (1936), Mehlberg (1980), Zeeman (1964), Winnie (1977), Malament (1977), Janis (1983).

14. As a fairly typical example we cite Prokhovnik (1967).

15. For a more complete discussion & explanation of the physical issues see Bethell (1990).

16. cp Grünbaum (1973) p. 827.

17. Cp. Nerlich (1982).

18. This was first pointed out by Varignon (1910)

19. But see also Bronner (1975) p. 114.

19 ~~18~~. The light sphere is not a Cauchy previous invariant hence the relativistic ~~of~~ stellar aberration. The motion of points in velocity space on the light space hold very naturally to the space representations of the Lorentz group.

21 ~~20~~ See Tangherlini (1961).

22 ~~21~~ Grünbaum (1973), Sjödin (1979), Salmon (1975).

4.3a Here is a very large literature addressed to the first possibility of measuring the one-way speed of light. For a representative refutation of a number of such referees see Salmon (1977).

23. For a clear discussion of the existence of this approach to the twin paradox, see, for example, Salmon (1975) p. 98.

24. See Filmer (1990) who adopts essentially this prescription. It synchronizes with a ray employing ~~the~~ radar rule along his trajectory.

25. A classic discussion along these lines is ~~Fuller~~ provided by Reichenbach (1958) p. 258 ff.

26 Putnam (1967).

Details on card index

References.

Kilmeister (1990) ✓

34

Earman (1989) ✓
Reichbach (1957) ✓
Grunbaum (1973) ✓
Eynstein (1905) ✓
Redhead (1983) ✓
Podlaha and Neustadtl (1966) ✓
Zahar (1977) ✓
Podlaha and Neustadtl (1967) ✓
Sjödin (1979) ✓
Torretti (1983) ✓
Friedman (1983) ✓
Grunbaum (1969a)
Quinn (1974) ✓
Rosenfeld (1975) ✓
Bridgman (1962) ✓
Friedman (1977) ✓
Grünbaum (1969a)
Robb (1914) ✓
Robb (1921) ✓
Robb (1936) ✓
Reichberg (1950) ✓
Zeeman (1964) ✓
Winnie (1977) ✓
Malament (1977) ✓
Soni (1983) ✓
Prokhovnik Prokhovnik (1967) ✓
Redhead (1990) ✓
Norton (1982) ✓
Vaneeth (1910) ✓
Brennan (1975) ✓
Tangherlini (1961) ✓
Salmon (1977) ✓
Salmon (1975) ✓